A short review of 'DGP spectroscopy'

2007 J. Phys. A: Math. Theor. 40 6827

(http://iopscience.iop.org/1751-8121/40/25/S26)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **40** (2007) 6827–6833 doi:10.1088/1751-8113/40/25/S26

A short review of 'DGP spectroscopy'

Antonio Padilla

School of Physics and Astronomy, University Park, University of Nottingham, Nottingham NG7 2RD, UK

E-mail: antonio.padilla@nottingham.ac.uk

Received 9 October 2006
Published 6 June 2007
Online at stacks.iop.org/JPhysA/40/6827

Abstract
In this paper we provide a short review of the main results developed in Charmousis *et al* (2006 *Preprint* hep-th/0604086). We focus on linearized vacuum perturbations about the self-accelerating branch of solutions in the DGP model. These are shown to contain a ghost in the spectrum for any value of the brane tension. We also comment on Deffayet *et al* (2006 *Preprint* hep-th/0607099), where some counter arguments have been presented.

PACS number: 11.25.−w

Recent observations of high redshift supernovae suggest that dark energy accounts for roughly 70% of the energy content of our universe [1]. This dark energy is consistent with a small positive cosmological constant, $\Lambda \sim 10^{-12}$ (eV)^4, exerting negative pressure on the universe, causing its expansion to accelerate. If we wish to resort to effective field theory methods to explain the origin of the cosmological constant, we typically run into an horrendous fine tuning problem. For a field theory cut off at the Planck scale, $m_{\rm pl}$, the natural value of the cosmological constant would be of the order $m_{\rm pl}^4$, which is 10^{120} orders of magnitude larger than the observed value.

This problem has inspired a search for alternative explanations of the current cosmic acceleration. One possibility is that it is due to new gravitational physics kicking in at the current Hubble scale, $H \sim 10^{-34}$ eV (see, for example, [2]). In this paper we will focus on the DGP model [3], which has arguably received more attention than any other model in which gravity is modified on ultra large scales. The model consists of a $\mathbb{Z}_2$ symmetric 3-brane embedded in 5D Minkowski space, described by the following action:

$$S = 2M_5^3 \int_{\rm bulk} \sqrt{-g}R + 4M_5^3 \int_{\rm brane} \sqrt{-\gamma}K + \int_{\rm brane} \sqrt{-\gamma}\left(M_4^2\mathcal{R} - \sigma + \mathcal{L}_{\rm matter}\right), \tag{1}$$

where g_{ab} is the bulk metric with corresponding Ricci tensor R_{ab}. The brane has induced metric $\gamma_{\mu\nu}$ with corresponding Ricci tensor $\mathcal{R}_{\mu\nu}$, and extrinsic curvature $K_{\mu\nu}$. The key feature here is the intrinsic curvature induced on the brane by matter loop corrections [4], or finite width effects [5]. Note that we have included an explicit brane tension σ, and additional matter

Lagrangian, $\mathcal{L}_{\text{matter}}$. The governing equations of motion in the bulk are simply the vacuum Einstein equations

$$G_{ab} = R_{ab} - \tfrac{1}{2} R g_{ab} = 0, \tag{2}$$

whereas the boundary conditions at the brane are given by the Israel junction conditions

$$\Theta_{\mu\nu} = 2M_5^3 (K_{\mu\nu} - K\gamma_{\mu\nu}) + M_4^2 \left(\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}\gamma_{\mu\nu} \right) + \frac{\sigma}{2}\gamma_{\mu\nu} = T_{\mu\nu}, \tag{3}$$

where $T_{\mu\nu} = -\frac{2}{\sqrt{-\gamma}} \frac{\partial(\sqrt{-\gamma}\mathcal{L}_{\text{matter}})}{\partial \gamma^{\mu\nu}}$. In the absence of any additional matter, we can set $T_{\mu\nu} = 0$, and derive the following background spacetimes:

$$\mathrm{d}s^2 = \bar{g}_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b = \mathrm{e}^{2\epsilon H|y|}(\mathrm{d}y^2 + \bar{\gamma}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu), \tag{4}$$

where $\epsilon = \pm 1$, and

$$\bar{\gamma}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -\mathrm{d}t^2 + \mathrm{e}^{2Ht}\,\mathrm{d}\vec{x}^2. \tag{5}$$

The bulk spacetime corresponds to $-\infty < y < 0$ and $0 < y < \infty$, with a de Sitter brane positioned at $y = 0$. The brane can be thought of as a 4D hyperboloid embedded in a 5D Minkowski bulk. The sign of ϵ determines whether the bulk spacetime corresponds to two copies of the exterior of the hyperboloid ($\epsilon = +1$), or two copies of the interior ($\epsilon = -1$). The solution with $\epsilon = -1$ is commonly referred to as the *normal* branch whereas the solution with $\epsilon = +1$ is referred to as the *self-accelerating* branch, a terminology which will become transparent shortly. Note that the metric in (5) represents the 4D de Sitter geometry in spatially flat coordinates, which covers only one half of the de Sitter hyperboloid.

The value of the intrinsic curvature on the brane can be related to the brane tension using the Israel equations (3). It turns out that

$$H = \frac{1}{2} H_0 \left(\epsilon + \sqrt{1 + \frac{\sigma}{3M_5^3 H_0}} \right), \tag{6}$$

where $H_0 = 2M_5^3 / M_4^2$ is taken to be the current Hubble scale. Note that even for vanishing tension, the self-accelerating solution gives rise to a de Sitter brane universe with $H = H_0$. The modification of gravity at large distances has enabled us to describe an accelerating universe in the absence of any vacuum energy whatsoever! In contrast, the normal branch gives rise to a Minkowski brane as $\sigma \to 0$, and is of less interest phenomenologically.

In 'DGP spectroscopy' [6], we discussed the stability of linearized perturbations about the background solution (4). On the normal branch, these perturbations are well behaved. In contrast, on the self-accelerating branch, one is generically haunted by ghosts. In this paper, we will review the discussion of linearized perturbations about the self-accelerating solution. For brevity, we will restrict attention to $\mathbb{Z}_2$ symmetric fluctuations about the vacuum ($T_{\mu\nu} = 0$). A more complete discussion including asymmetric fluctuations and the contribution from additional matter ($T_{\mu\nu} \neq 0$) can be found in [6].

Recall that the self-accelerating background solution is given by the metric (4) with $\epsilon = +1$ and the brane positioned at $y = 0$. A generic perturbation can be described by $g_{ab} = \bar{g}_{ab} + \delta g_{ab}$ with the brane position shifted to $y = F(x)$. It is convenient to work in a Gaussian normal (GN) gauge so that

$$\delta g_{yy} = \delta g_{\mu y} = 0, \qquad \delta g_{\mu\nu} = \mathrm{e}^{H|y|/2} h_{\mu\nu}(x, y). \tag{7}$$

The tensor $h_{\mu\nu}$ can be decomposed in terms of the irreducible representations of the 4D de Sitter diffeomorphism group

$$h_{\mu\nu} = h_{\mu\nu}^{\mathrm{TT}} + D_\mu A_\nu + D_\nu A_\mu + D_\mu D_\nu \phi - \frac{1}{4}\bar{\gamma}_{\mu\nu} D^2 \phi + \frac{h}{4}\bar{\gamma}_{\mu\nu}, \tag{8}$$

where D_μ is the covariant derivative for the 4D de Sitter metric $\bar{\gamma}_{\mu\nu}$. The transverse-tracefree tensor $h^{\text{TT}}_{\mu\nu}$ satisfies $D^\mu h^{\text{TT}}{}_{\mu\nu} = h^{\text{TT}\mu}{}_\mu = 0$, and has five independent components. A_μ is a Lorentz-gauge vector, $D^\mu A_\mu = 0$, with three independent components, and ϕ and $h = h^\mu{}_\mu$ are two scalar fields[1].

We can fix the position of the brane to be at $y = 0$, whilst remaining in GN gauge, by making the following gauge transformation:

$$y \to y - F\,\mathrm{e}^{-H|y|}, \qquad x^\mu \to x^\mu - \frac{\mathrm{e}^{-H|y|}}{H} D^\mu F. \tag{9}$$

Although the brane position is now fixed at $y = 0$, the original brane position $F(x)$ still enters the dynamics through a book-keeping term $h^{(F)}_{\mu\nu}$ that modifies the metric perturbation

$$h_{\mu\nu} \to h^{\text{TT}}_{\mu\nu} + D_\mu A_\nu + D_\nu A_\mu + D_\mu D_\nu \phi - \frac{1}{4}\bar{\gamma}_{\mu\nu} D^2\phi + \frac{h}{4}\bar{\gamma}_{\mu\nu} + h^{(F)}_{\mu\nu}. \tag{10}$$

The book-keeping term is of course pure gauge in the bulk, and is given by

$$h^{(F)}_{\mu\nu} = \frac{2}{H}\,\mathrm{e}^{H|y|/2}(D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})F. \tag{11}$$

We can now substitute our modified expression for $h_{\mu\nu}$ into the linearized fields equations in the bulk, $\delta G_{ab} = 0$, and on the brane, $\delta\Theta_{\mu\nu} = 0$. It turns out that the Lorentz-gauge vector A_μ is a free field in the linearized theory and can be set to zero. In addition, the yy and $y\mu$ equations in the bulk imply that one can consistently choose a gauge for which

$$h = 0, \qquad (D^2 + 4H^2)\phi = 0. \tag{12}$$

Note that we now have $h_{\mu\nu} = h^{\text{TT}}_{\mu\nu} + h^{(\phi)}_{\mu\nu} + h^{(F)}_{\mu\nu}$, where the contribution from $\phi(x, y)$ has been rewritten as follows:

$$h^{(\phi)}_{\mu\nu} = (D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})\phi(x, y). \tag{13}$$

This mode is now entirely transverse tracefree in its own right. In the absence of any additional matter on the brane ($T_{\mu\nu} = 0$), the same is true of the bookkeeping mode, $h^{(F)}_{\mu\nu}$. This is because the trace of the Israel equation now implies that

$$(D^2 + 4H^2)F = 0. \tag{14}$$

The entire perturbation $h_{\mu\nu}(x, y)$ is now completely transverse tracefree. This greatly simplifies the bulk and brane equations of motion, giving

$$\left[D^2 - 2H^2 + \partial_y^2 - \frac{9H^2}{4}\right] h_{\mu\nu}(x, y) = 0 \qquad \text{for } |y| > 0 \tag{15}$$

$$\left[M_4^2(D^2 - 2H^2) + 2M_5^3\left(\partial_y - \frac{3H}{2}\right)\right] h_{\mu\nu} = 0 \qquad \text{at } y = 0^+. \tag{16}$$

We now separate variables in the tensor and scalar fields as follows:

$$h^{\text{TT}}_{\mu\nu}(x, y) = \sum_m u_m(y)\chi^{(m)}_{\mu\nu}(x), \qquad \phi(x, y) = W(y)\hat{\phi}(x), \tag{17}$$

where $\chi^{(m)}_{\mu\nu}$ is 4D tensor field of mass m satisfying $(D^2 - 2H^2)\chi^{(m)}_{\mu\nu} = m^2\chi^{(m)}_{\mu\nu}$. Note that $\hat{\phi}$ is a 4D tachyonic field satisfying $(D^2 + 4H^2)\hat{\phi} = 0$. This is a mild instability which is related to the repulsive nature of inflating domain walls.

We shall now focus on the case where the brane tension is non-vanishing ($\sigma \neq 0$). Assuming that the tensor and scalar equations of motion can be treated independently, we find

[1] Note that the total number of independent components correctly adds up 10.

that there is a continuum of normalizable tensor modes with mass $m^2 \geqslant 9H^2/4$. In addition, there is also a discrete tensor mode with mass

$$m_d^2 = H_0(3H - H_0) \tag{18}$$

and normalizable wavefunction $u_{m_d}(y) = \alpha_{m_d}\,\mathrm{e}^{-|y|\sqrt{\frac{9H^2}{4}-m_d^2}}$. Now, for *positive* brane tension $\sigma > 0$, one can easily check that $0 < m_d^2 < 2H^2$. For massive gravitons propagating in 4D de Sitter, it is well known that masses lying in this range result in the graviton containing a helicity-0 ghost [7]. This means that for $\sigma > 0$, the lightest tensor mode contains a helicity-0 ghost, and so the system is perturbatively unstable. For *negative* brane tension, $m_d^2 > 2H^2$ and there is no helicity-0 ghost in the lightest tensor.

Now consider the scalar equations of motion. The first thing to note is that $h^{(\phi)}_{\mu\nu}$ behaves like a transverse-tracefree mode with mass $m_\phi^2 = 2H^2$, because $(D^2 - 2H^2)h^{(\phi)}_{\mu\nu} = 2H^2 h^{(\phi)}_{\mu\nu}$. Since none of the tensor modes have this mass, they are all orthogonal to $h^{(\phi)}_{\mu\nu}$. This means it was consistent to assume that the scalar and tensor equations of motion could indeed be treated independently. It turns out that the scalar has a *normalizable* wavefunction $W(y) = \mathrm{e}^{-H|y|/2}$, and the 4D scalar $\hat{\phi}$ is sourced by F via the relation

$$\hat{\phi}(x) = \alpha F(x), \qquad \alpha = -\left[\frac{2H - H_0}{H(H - H_0)}\right] \tag{19}$$

This is well defined for $\sigma \neq 0$ since then $H \neq H_0$. $h^{(\phi)}_{\mu\nu}(x, y)$ may now be thought of as a genuine radion mode, measuring the physical motion of the brane with respect to infinity. It does *not* decouple even though we only have a single brane. This property is related to the fact that the background warp factor, $\mathrm{e}^{2H|y|}$, *grows* as we move deeper into the bulk.

We have already identified the helicity-0 mode of the lightest tensor as a ghost when $\sigma > 0$. When $\sigma < 0$, a calculation of the 4D effective action will reveal the ghost to be the radion. Taking our solution

$$\begin{aligned} h_{\mu\nu}(x, y) = {} & \alpha_{m_d}\,\mathrm{e}^{-|y|\sqrt{\frac{9H^2}{4}-m_d^2}}\chi^{(m_d)}_{\mu\nu}(x) + \alpha\,\mathrm{e}^{-H|y|/2}(D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})F \\ & + \frac{2}{H}\,\mathrm{e}^{H|y|/2}(D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})F + \text{continuum modes} \end{aligned} \tag{20}$$

and inserting it into action (1), we can integrate out the extra dimension. This is made possible by restricting attention to normalizable modes. The result is $S_{\text{eff}} = \int \mathrm{d}^4x\sqrt{-\bar{\gamma}}\mathcal{L}_{\text{eff}}$, where

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{PF}}[\chi^{(m_d)}] - \frac{m_d^2}{4}\chi^{(m_d)\mu\nu}\left(\chi^{(m_d)}_{\mu\nu} - \chi^{(m_d)}\bar{\gamma}_{\mu\nu}\right) - 3M_5^3H^2\alpha F(D^2 + 4H^2)F + \cdots \tag{21}$$

$\mathcal{L}_{\text{PF}}$ is the standard Pauli–Fierz Lagrangian and '$\cdots$' denotes the contribution from the continuum of tensor modes. When $\sigma < 0$, it turns out that $\alpha > 0$, and so we confirm that negative brane tension gives rise to a radion ghost.

Given that there is always a ghost for non-zero tension, one might expect by continuity that this remains the case when $\sigma = 0$. To study this more closely, let us first ask whether we can trust the above solutions in the limit where $\sigma \to 0$. In this limit $H \to H_0$, and the quantity α becomes ill defined! To understand what has gone wrong, note that the mass of the lightest tensor $m_d^2 \to 2H^2$. This means that it is no longer orthogonal to the radion, $h^{(\phi)}_{\mu\nu}$, and so we cannot treat the tensor and scalar equations of motion independently. This behaviour can be traced back to an additional symmetry that appears in the linearized theory in the limit of vanishing brane tension. It is analogous to the 'partially massless limit' in the theory of a massive graviton propagating in de Sitter space [7]. In *that* theory, the equations of motion are invariant under the following redefinition of the graviton field:

$$\chi^{(\sqrt{2}H)}_{\mu\nu}(x) \to \chi^{(\sqrt{2}H)}_{\mu\nu}(x) + (D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})\psi(x). \tag{22}$$

This field redefinition has the effect of extracting out part of the helicity-0 mode from $\chi_{\mu\nu}^{(\sqrt{2}H)}$, and as a result of the symmetry this mode disappears from the spectrum. In *our* case, this shift must be accompanied by a shift in the scalar field ϕ,

$$\phi(x, y) \to \phi(x, y) - \alpha_{\sqrt{2}H}\, \mathrm{e}^{-H|y|/2}\psi(x) = \phi(x, y) - \lim_{\sigma\to 0} \alpha_{m_d}\, \mathrm{e}^{-|y|\sqrt{\frac{9H^2}{4}-m_d^2}}\psi(x) \tag{23}$$

in order to render the overall perturbation, $h_{\mu\nu}(x, y)$, invariant. These ψ shifts can be understood as the extracting part of the helicity-0 mode from $\chi_{\mu\nu}^{(\sqrt{2}H)}$ and absorbing it into a renormalization of ϕ. The symmetry will have the effect of combining the helicity-0 mode and the radion into a single degree of freedom. It is only *after* fixing this ψ symmetry that we can treat the scalar and tensor equations of motion independently of one another. We might think of extracting the entire helicity-0 mode and absorbing it into ϕ, or vice versa. Actually, it will be convenient to make a different gauge choice that enables us to take a smooth limit as $\sigma \to 0$ [8]. We start off with the solution for $\sigma \neq 0$ given by equation (20), and make the field redefinition

$$\chi_{\mu\nu}^{(m_d)} \to \mathcal{H}_{\mu\nu} = \chi_{\mu\nu}^{(m_d)} + (D_\mu D_\nu + H^2\gamma_{\mu\nu})\left[\frac{\alpha}{\alpha_{m_d}}F\right]. \tag{24}$$

In the limit as $\sigma \to 0$, this has the effect of extracting out part of the helicity-0 mode of $\chi_{\mu\nu}^{(m_d)}$ and absorbing it into a renormalization of ϕ:

$$\phi(x, y) \to \lim_{\sigma\to 0} \mathrm{e}^{-H|y|/2}\alpha F - \alpha_{m_d}\, \mathrm{e}^{-|y|\sqrt{\frac{9H^2}{4}-m_d^2}}\left[\frac{\alpha}{\alpha_{m_d}}F\right] = -|y|\, \mathrm{e}^{-H|y|/2}F. \tag{25}$$

It follows that for vanishing tension

$$\begin{aligned} h_{\mu\nu}(x, y) = {} & \mathrm{e}^{-H|y|/2}[\alpha_{\sqrt{2}H}\mathcal{H}_{\mu\nu} - |y|(D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})F] \\ & + \frac{2}{H}\,\mathrm{e}^{H|y|/2}(D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})F + \text{continuum modes}, \end{aligned} \tag{26}$$

where the 4D tensor $\mathcal{H}_{\mu\nu}$ satisfies

$$\alpha_{\sqrt{2}H}(D^2 - 4H^2)\mathcal{H}_{\mu\nu} = -H(D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})F. \tag{27}$$

This equation should be understood as the helicity-0 component of $\mathcal{H}_{\mu\nu}$ being completely determined by the source F. A calculation of the 4D effective action in this case, now gives

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{PF}}[\mathcal{H}] - \frac{H^2}{2}\mathcal{H}^{\mu\nu}(\mathcal{H}_{\mu\nu} - \mathcal{H}\bar{\gamma}_{\mu\nu}) + \frac{\sqrt{2}M_5^3}{M_4}\mathcal{H}^{\mu\nu}(D_\mu D_\nu + H^2\bar{\gamma}_{\mu\nu})F + \cdots. \tag{28}$$

A derivation of the Hamiltonian for this action reveals the presence of a scalar degree of freedom whose energy is unbounded from below [8]. This ghost is a combination of the radion and helicity-0 mode, and represents the residual scalar degree of freedom left over after fixing the aforementioned ψ symmetry.

We conclude that for any value of the brane tension, perturbations about the self-accelerating branch of DGP contain a ghost. We would like to emphasize that this ghost-like instability is ultimately *classical*, and one cannot hide behind a UV completion of DGP to save the day. The ghost couples to matter, and even in the absence of matter we would expect it to couple to the tensor modes through higher order interactions. Given that its energy is unbounded from below, the ghost will continually dump its energy into the other fields, rapidly destroying the self-accelerating solution. We might expect the rate of the instability to go like the frequency of oscillation of the coupled fields. Given that we have an entire tower of heavy-tensor modes, this frequency could be very large indeed.

We would like to end our discussion with a few comments on [9], where it has been argued that the field F in action (28) is nothing more than a Lagrange multiplier and can be consistently set to zero. This has the effect of eliminating the scalar ghost from the spectrum. However, it is important to realize what it *really* means to set $F = 0$ in (28), from the point of view of the full bulk solution. To eliminate F from (28), we need to introduce a *non-normalizable* mode in $\phi(x, y)$ that cancels off the contribution from the bookkeeping term $h^{(F)}_{\mu\nu}$. To see how this works, consider the general bulk solution for $\phi(x, y)$, retaining both normalizable and non-normalizable modes. This is given by

$$\phi(x, y) = \mathrm{e}^{-H|y|/2}\hat{\phi}(x) + \mathrm{e}^{H|y|/2}\tilde{\phi}(x), \tag{29}$$

where the last term corresponds to the non-normalizable mode. In our analysis, we invoked the condition of normalizability to set $\tilde{\phi}(x) = 0$, regardless of the value of the brane bending term F. In order to set F to zero in (28), as suggested in [9], we need to impose the boundary condition $\tilde{\phi} = -2F/H$. This boundary condition seems to violate a local, causal 4D description on the brane: if we change F ever so slightly, we need the boundary condition for $\phi(x, y)$ as $y \to \infty$ to respond accordingly! Even if we accept this, there is strong evidence to suggest that retaining non-normalizable modes in the spectrum leads to further problems with ghosts. This was discussed in [6] in the context of the gravitational field of a relativistic particle, or 'shockwave', on the brane. The non-normalizable modes contribute a *repulsive* potential on the brane, which would indicate the presence of ghosts.

It is also argued in [9] that in the presence of a heavy source, linearized perturbation theory breaks down below a Vainshtein radius r_V, and so one cannot make any conclusions as to whether or not the ghost is really there. Even in the region $r \gg r_V$ where the linearized theory makes sense, it is not obvious that the linearized solution can always be smoothly continued inside r_V. If this is indeed the case, one cannot use perturbations about the self-accelerating background (4) to make any reliable cosmological predictions. To proceed, we need to identify background solutions that take into account localized brane sources. To our knowledge, the only known exact solution with a localized brane source is the shockwave [10]. It would be interesting to study perturbation theory about this solution in order to see if the ghost remains.

Acknowledgments

I would like to thank C Charmousis, R Gregory and N Kaloper for a fruitful collaboration, as well as the Universitat de Barcelona, where most of this work was completed.

References

[1] Riess A G *et al* (Supernova Search Team Collaboration) 1998 *Astron. J.* **116** 1009 (*Preprint* astro-ph/9805201)
Riess A G *et al* 2004 *Preprint* astro-ph/0402512
Perlmutter S *et al* (Supernova Cosmology Project Collaboration) 1999 *Astrophys. J.* **517** 565 (*Preprint* astro-ph/9812133)
[2] Arkani-Hamed N, Dimopoulos S, Dvali G and Gabadadze G 2002 *Preprint* hep-th/0209227
Kogan I I, Mouslopoulos S, Papazoglou A, Ross G G and Santiago J 2000 *Nucl. Phys.* B **584** 313 (*Preprint* hep-ph/9912552)
Kogan I I and Ross G G 2000 *Phys. Lett.* B **485** 255 (*Preprint* hep-th/0003074)
Gregory R, Rubakov V A and Sibiryakov S M 2000 *Phys. Rev. Lett.* **84** 5928 (*Preprint* hep-th/0002072)
Gregory R, Rubakov V A and Sibiryakov S M 2000 *Phys. Lett.* B **489** 203 (*Preprint* hep-th/0003045)
Padilla A 2005 *Class. Quantum Grav.* **22** 681 (*Preprint* hep-th/0406157)
Padilla A 2005 *Class. Quantum Grav.* **22** 1087 (*Preprint* hep-th/0410033)
[3] Dvali G R, Gabadadze G and Porrati M 2000 *Phys. Lett.* B **485** 208 (*Preprint* hep-th/0005016)
Dvali G R and Gabadadze G 2001 *Phys. Rev.* D **63** 065007 (*Preprint* hep-th/0008054)
Deffayet C 2001 *Phys. Lett.* B **502** 199 (*Preprint* hep-th/0010186)
Deffayet C, Dvali G R and Gabadadze G 2002 *Phys. Rev.* D **65** 044023 (*Preprint* astro-ph/0105068)
Lue A 2005 *Preprint* astro-ph/0510068

[4] Birrell N D and Davies P C W 1982 *Quantum Fields in Curved Space* (Cambridge: Cambridge University Press)
[5] Carter B and Gregory R 1995 *Phys. Rev.* D **51** 5839 (*Preprint* hep-th/9410095)
Bonjour F, Charmousis C and Gregory R 2000 *Phys. Rev.* D **62** 083504 (*Preprint* gr-qc/0002063)
Kiritsis E, Tetradis N and Tomaras T N 2001 *J. High Energy Phys.* JHEP08(2001)012 (*Preprint* hep-th/0106050)
[6] Charmousis C, Gregory R, Kaloper N and Padilla A 2006 *Preprint* hep-th/0604086
[7] Deser S and Nepomechie R I 1984 *Ann. Phys.* **154** 396
Higuchi A 1987 *Nucl. Phys.* B **282** 397
Higuchi A 1989 *Nucl. Phys.* B **325** 745
Kogan I I, Mouslopoulos S and Papazoglou A 2001 *Phys. Lett.* B **503** 173 (*Preprint* hep-th/0011138)
Deser S and Waldron A 2001 *Phys. Rev. Lett.* **87** 031601 (*Preprint* hep-th/0102166)
Bengtsson I 1995 *J. Math. Phys.* **36** 5805 (*Preprint* gr-qc/9411057)
Deser S and Waldron A 2001 *Phys. Lett.* B **508** 347 (*Preprint* hep-th/0103255)
[8] Koyama K 2005 *Phys. Rev.* D **72** 123511 (*Preprint* hep-th/0503191)
Gorbunov D, Koyama K and Sibiryakov S 2005 *Preprint* hep-th/0512097
[9] Deffayet C, Gabadadze G and Iglesias A 2006 *Preprint* hep-th/0607099
[10] Kaloper N 2005 *Phys. Rev.* D **71** 086003
Kaloper N 2005 *Phys. Rev.* D **71** 129905 (*Preprint* hep-th/0502035) (erratum)